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Measuring host rock volume changes during magma emplacement: Discussion

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1. Introduction

In the paper under discussion, Yoshinobu and Girty (1999) have illustrated a potentially very useful technique for measuring volume change, which has obvious applicability to the 'space problem' that arises in pluton emplacement. Since the numerical estimates given in their paper are liable to be referenced widely in this field of research, it is necessary to draw attention to errors in the confidence limits quoted by the authors, the use of which give the impression that their estimated values are more precise than is justified. In their study of the contact aureole of the Emigrant Gap composite pluton (Sierra Nevada, California) they have, for instance, reported the percentage change in total rock mass between chlorite grade rocks respectively subjected to, and unaffected by, the thermal influence of the pluton and aureole as $-11.1\% \pm 1.4\%$ (presumably as 95% confidence limits, since their other values are thus quoted), on the assumption that aluminium is an immobile reference frame element. It is suggested below that the standard error of the mean is 3.6% and thus the 95% confidence limits are more nearly $\pm 7\%$.

2. Theory

Yoshinobu and Girty use three equations (see Ague, 1991; Brimhall and Dietrich, 1987). Quoted as fractional, rather than percentage, changes, the first gives the fractional mass change from an initial to a final state, T_i, based on an immobile reference-frame element i:

$$T_i = (C_i^0 / C_i') - 1, \tag{1}$$

where C_i^0 and C_i' are, respectively, the concentrations of the reference-frame element i outside and inside the contact aureole (initial and final states). The second equation describes the fractional mass change of a specific mobile species j relative to a fixed, immobile species i:

$$\tau_{ji} = (C_i^0 / C_i')(C_j' / C_j^0) - 1. \tag{2}$$

The third equation gives the fractional volume strain:

$$\varepsilon_i = (\rho^0 / \rho')(C_i^0 / C_i') - 1, \tag{3}$$

where ρ^0 and ρ' are the corresponding initial and final densities.

From these equations it is clear that we are dealing with multivariate functions, specifically $T_i(C_i^0, C_i')$, $\tau_{ji}(C_i^0, C_i', C_j^0, C_j')$ and $\varepsilon_i(\rho^0, \rho', C_i^0, C_i')$. For a function $z = f(x, y, \ldots)$, provided that the measured quantities $x, y, \ldots$ are independent and errors in them are uncorrelated, the law of propagation of errors allows us to relate the variance in z to variances in the measured quantities (see, for example, Meyer, 1975, pp. 40, 41):

$$S_z^2 = (\partial f / \partial x)^2 S_x^2 + (\partial f / \partial y)^2 S_y^2 + \ldots \tag{4}$$

Applying this to the previous equations we derive:

$$S_{T_i}^2 = (T_i + 1)^2 \left\{ (S_{C_i^0})^2 / C_i^{0\,2} + (S_{C_i'})^2 / C_i'^{\,2} \right\}; \tag{5}$$

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$$S^2_{\tau_{ji}} = (\tau_{ji}+1)^2\Big\{(S_{C^0_i})^2/C^{0\,2}_i + (S_{C'_i})^2/C'^{\,2}_i + (S_{C^0_j})^2/C^{0\,2}_j + (S_{C'_j})^2/C'^{\,2}_j\Big\}; \qquad (6)$$

$$S^2_{\varepsilon_i} = (\varepsilon_i+1)^2\Big\{(S^2_{\rho^0})^2/\rho^{02} + (S^2_{\rho'})^2/\rho'^{\,2} + (S_{C^0_i})^2/C^{0\,2}_i + (S_{C'_i})^2/C'^{\,2}_i\Big\}. \qquad (7)$$

Yoshinobu and Girty have argued the case for treating Al as an immobile reference element. They have therefore applied Eq. (1) to Al. From their table 1:

$C^0_{Al} = 20.0 \pm 1.59\%$ on $13-1=12$ degrees of freedom;

$C'_{Al} = 22.5 \pm 0.99\%$ on $26-1=25$ degrees of freedom,

quoting 95% confidence limits. Since the 95% confidence level is $t_{0.05,\,n-1}$ times the standard error of the mean, the corresponding standard errors are:

$S_{C^0_{Al}} = 0.48\%$;

$S_{C'_{Al}} = 0.73\%$.

From Eq. (1):

$T_{Al} = -0.111$

and, from Eq. (5):

$S_{T_{Al}} = 0.036$.

The minimum value for the factor relating the standard error and the 95% confidence limit is 1.96, in the case where the standard error is known, rather than estimated from a sample of limited size. The 95% confidence level in T_{Al} will therefore be at least twice the estimated standard error, leading to a result of $T_{Al} = -11 + 7\%$. (A very crude, but quick, estimate of the standard error in T_{Al} can be made directly from Eq. (1). If we average the standard errors $S_{C^0_{Al}}$ and $S_{C'_{Al}}$ at about 0.6%, take the standard error of a difference as $\sqrt{2}$ times the individual standard error, and ignore the error in the denominator, we derive 3.8%, which is of the same order as the more precisely calculated value.)

Yoshinobu and Girty use Eq. (2) to calculate the amount of SiO_2 transferred out of the aureole during contact metamorphism. Proceeding as above we can calculate the standard errors in the initial and final concentrations of Si as $S_{C^0_{Si}} = 1.16\%$ and $S_{C'_{Si}} = 0.68\%$, and thus, from Eq. (6), the calculated standard error in τ_{SiAl} is 3.8%, which is equivalent to 95% limits of about 7.7%.

To calculate the fractional volume strain requires, additionally, knowledge of initial and final densities. Yoshinobu and Girty do not quote errors in their density measurements, but it is clear from a comparison of Eqs. (7) and (5) that the error limits on volume strain will be similar to those on T_{Al}, so that the volume strain estimated from their figures is about $-12 \pm 7\%$ at the 95% level.

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